OPTIMIZING SOLUTION PROCEDURES FOR THE TRANSSHIPMENT PROBLEM WITH MIXED CONSTRAINTS

A Synopsis

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Introduction

Transshipment problem is a generalisation of the transportation problem. We consider a generalisation of the transshipment problem in which the origin and destination constraints consists not only of equality but also of greater than or equal to or less than or equal to type constraints.

Transportation problem is to select the most economical routes for transfer of a uniform (homogeneous) commodity from a number of sources to a number of destinations while satisfying supply and demand limits. That is, the problem is to find the amount of a uniform commodity which should be transported from each m sources to each n destination satisfying all the supply and demand limits of sources and destinations respectively so that the overall transporting cost is minimum. Note that per unit transporting cost of commodity from each source to each destination is given. If the supply and demand limits of individual sources and destinations are fixed (constant) then the problem is said to be the transportation problem with equality constraints or a classical transportation problem.

In classical transportation problem shipment of homogeneous commodity takes place from source to destination such as from factories to warehouses, or from warehouses to supermarkets. But instead of direct shipments from sources to destinations, the commodity can be transported to a particular destination through one or more intermediate source and destination. Each of these points in turn supply to other points. Thus when the shipments pass from destination to destination and from source to source also, then minimizing the overall transshipment cost satisfying the demand and supply limits of sources and destinations is said to be a transshipment problem or a classical transshipment problem. In the transshipment problem all the sources and destinations can function in any direction, i.e., from destination to destination, source to source, source to destination, destination to source also.

If the capacity of a source is substantially increased / decreased and the requirement of a destination is also substantially increased / decreased, how will affect the overall transshipment cost? Considering this fact the transshipment problem slightly differs from the classical transshipment problem. Thus in real life, however, most transshipment problems have mixed constraints accommodating many applicatons that go beyond transshipment problem with equality constraints.

In the classical transshipment problem availabilities at the sources and the requirements at the destinations are fixed quantities and met exactly by the solution. In transshipment problem with mixed constraints the availability and / or requirement constraints were inequations which opposed to the usual equations. That is, availability constraints and requirement constraints are of mixed type. The problem having such type of constraints is said to be the transshipment problem with mixed constraints.

In many applications shipments are allowed between supply points or between demand points to reduce the total transporting cost. Usually, in the absence of the transshipment, the transportation cost goes higher. Hence the transshipment is also very useful to reduce the transportation cost.

Transshipment problem with mixed constraints can be seen mathematically with the help of the following terminologies and transportation table.

O_i = i^{th} source(origin), i =1,2,...m

D_j = j^{th} destination, j =1,2,...n

m = number of sources

n = number of destinations

a_i = quantity of uniform (homogeneous) commodity available at source O_i

b_j = quantity of uniform (homogeneous) commodity required at destination D_j

C_{ij} = per unit transshipment cost of commodity from source O_i to destination D_j.

x_{ij} = quantity of commodity shipped from origin O_i to destination D_j

Transportation Table

	D_1	D_2	……	D_j	……	D_n	supply
O_1	x_{11} c_{11}	x_{12} c_{12}	……	x_{1j} c_{1j}	……	x_{1n} c_{1n}	a_1
O_2	x_{21} c_{21}	x_{22} c_{22}	……	x_{2j} c_{2j}	……	x_{2n} c_{2n}	a_2
……	……	……	……	……	……	……	……
O_i	x_{i1} c_{i1}	x_{i2} c_{i2}	……	x_{ij} c_{ij}	……	x_{in} c_{in}	a_i
……	……	……	……	……	……	……	……
O_m	x_{m1} c_{m1}	x_{m2} c_{m2}	……	x_{mj} c_{mj}	……	x_{mn} c_{mn}	a_m
Demand	b_1	b_2	……	b_j	……	b_n	

Since any source and destination can ship to any other origin and destination, it would be convenient to number them successively so that the origins are numbered from 1 to m and the destinations from m+1 to m+n. Now we have m+n sources $O_1, O_2, ..., O_m, D_1, D_2, ..., D_n$ with supply $a_1, a_2, ...a_m, 0, 0, ..., 0$ and m+n destinations $O_1, O_2, ..., O_m, D_1, D_2, ..., D_n$ with demands $0, 0, ..., 0, b_1, b_2, ...b_n$. Note that $c_{ij} = 0, \ \forall \ i = j = 1, 2, ...m + n$.

The total amount shipped from an origin must be equal to the amount it produces plus what it transships. Similarly, the total amount received at a destination must be equal to its demand plus what it transships.

Thus, the mathematical formulation of transshipment problem with mixed constraints is to find x_{ij} which

$$\text{Min z} = \sum_{i=1}^{m+n} \sum_{j=1, j\neq i}^{m+n} c_{ij} x_{ij}$$

$$\text{such that} \begin{cases} \sum_{j=1, j\neq i}^{m+n} x_{ij} - \sum_{j=1, j\neq i}^{m+n} x_{ji} \geq a_i, \ i \in Q \\ \sum_{j=1, j\neq i}^{m+n} x_{ij} - \sum_{j=1, j\neq i}^{m+n} x_{ji} \leq a_i, \ i \in T \\ \sum_{j=1, j\neq i}^{m+n} x_{ij} - \sum_{j=1, j\neq i}^{m+n} x_{ji} = a_i, \ i \in S \\ \sum_{i=1, i\neq j}^{m+n} x_{ij} - \sum_{i=1, i\neq j}^{m+n} x_{ji} \geq b_j, \ \text{j} \in \text{U} \\ \sum_{i=1, i\neq j}^{m+n} x_{ij} - \sum_{i=1, i\neq j}^{m+n} x_{ji} \leq b_j, \ \text{j} \in \text{V} \\ \sum_{i=1, i\neq j}^{m+n} x_{ij} - \sum_{i=1, i\neq j}^{m+n} x_{ji} = b_j, \ \text{j} \in \text{W} \\ x_{ij} \geq 0, \ i = 1, 2, ...m+n, \ j = 1, 2, ...m+n, \ j \neq i, \ \text{and integers} \end{cases}$$

where Q, T and S are pairwise disjoint subsets of {1,2,.....m+n} such that $Q \cup T \cup S = \{1, 2, ...m+n\}$;

U, V and W are pairwise disjoint subsets of {1,2,...m+n} such that $U \cup V \cup W = \{1, 2, ...m+n\}$;

If Q, T, U and V are empty, then the problem becomes the transshipment problem with equality constraints. i.e., classical transshipment problem.

Unbalanced transshipment problem with mixed constraints arises when

$$\sum_{i=1}^{m+n} a_i > \sum_{j=1}^{m+n} b_j$$

or when

$$\sum_{i=1}^{m+n} a_i < \sum_{j=1}^{m+n} b_j$$

It can be balanced by introducing dummy transshipment point having transshipment cost zero.

Survey of Published Research Work

We have gone through a number of research papers related to transportation and transshipment problem. Out of them some of the recent works cited below are the source of inspiration to do the research work.

Transportation problem with mixed constraints was rigorously studied firstly by Brigden(1974)[3]. He solved this problem by considering a related standard transportation problem having two additional sources and two additional destinations.

Gupta et al.(1993)[5], also studied transportation problem with mixed constraints.

Arora and Khurana(2002)[2], studied paradox in indefinite quadratic transportation problem with mixed constraints.

Later Adlakha et al.(2006)[1], developed a heuristic for solving the transportation problem with mixed constraints.

Later Khurana et al.(2009,2010)[10,11], solved quadratic transportation problem for the case of restricted flow in 2009 and found out cost-time trade pairs for quadratic transportation problem in 2010.

Williams(1986)[22], developed a method called the Fourier elimination method for solving linear programming problems based on the Fourier elimination technique which generates more additional and redundant constraints.

Kanniappan and Thangavel(1998)[9], modified the method of Fourier elimination method developed by Williams[22], for solving linear programming problems by choosing a variable for elimination, which generates minimum number of constraints in each step compared to that of the method of Fourier elimination method.

Pandian and Natrajan(2010a)[17], have developed Fourier method for solving transportation problems with mixed constraints. They converted all the constraints in less than or equal to ($\leq$) type constraints and eliminated variables one by one with addition of a specified constraints. Later on, their another paper (2010b)[18], presented another method for finding optimal more-for-less solution of transportation problem with mixed constraints. More-for-less solution means more supply on less transporting cost.

Transshipment problem was first introduced by Orden(1956)[15]. He gave an extension of the original transportation problem to include the possibility of transshipment. That is, any shipping or receiving point is also permitted to act as an intermediate point. The transshipment technique is used to find the shortest route from one point in a network to another.

The problem of determining simultaneously the flows of primary products through processors to the market as final products has been formulated alternatively as a transhipment model by King and Logan (1964)[13], and as a reduced matrix model by Rhody(1963)[20]. An extension of this problem to a multiregional, multiproduct and multiplant problem, formulated in the form of general linear programming model, has been proposed by Judge et al.(1965)[8]. Afterwards, various alternatives formulations of the transshipment problem within the framework of the transportation model that permits solution of problems of the type discussed by King and Logan without the need of subtraction of artificial variables were discussed by Hurt and Tramel(1965)[7]. Garg and Prakash(1985)[4], studied time minimizing transshipment problem. Later, dynamic transshipment problem was studied by Herer and Tzur(2001)[6].

Afterwards, Multilocation transshipment problem with capacitated production and lost sales was studied by Ozdemir(2006)[16].

Khurana and Arora(2011)[12], have solved transshipment problem with mixed constraints by transforming the problem into an equivalent transportation problem by introducing an additional row and a column.

Dual matrix approach is developed by Ping and CHU(2002)[19], to solve transportation problem with inequality constraints. They converted the problem into corresponding dual problem and after using a sequence of matrix operations, optimal solution is obtained.

Sudhakar et.al.(2012)[21], developed a method for finding an optimal solution of a transportation problem. The idea introduced in their paper is a new direction for searching the optimal solution of the transportation problem. They obtain at least one zero in each row and each column of the transportation table by subtracting the least element of each row / column from all the elements of the corresponding row / column. Get suffix value of each zero and assign the cell corresponding to the greatest suffix value. Delete the exhausted row / column

to get the reduced table. Procedures are repeated for the reduced table until all the demands and supplies are exhausted.

Proposed Work

A literature search revealed that much effort has been concentrated on classical transportation problem as well as transshipment problem with equality constraints. Both the transportation and transshipment problems are computationally easy to solve. Only one paper [12], as observed in the literature, addresses the transshipment problem with mixed constraints. No systematic method has been developed for finding an optimal solution or addressing more-for-less situations in transshipment problems with mixed constraints. Proper and efficient methods should be developed for solving the transshipment problem with mixed constraints.

We are going to extend the transportation problem with mixed constraints to permit transshipment with the additional feature that shipments may go via any sequence of points rather than being restricted to direct connections from one origin to one destination. We propose to develop some methods for solving the transshipment problem with mixed constraints.

The idea used in the paper [12], motivated us to develop some more efficient methods for solving transshipment problem with mixed constraints. In this paper, the problem is converted into equivalent standard transshipment problem by introducing two dummy transshipment points. Note that the transshipment point is nothing but either a source or a destination.

Since the transshipment problem with mixed constraints is the generalisation of transportation problem with mixed constraints, the technique developed so far for the transportation problem with mixed constraints, as addressed in the literature, may be extended after some modifications where ever necessary to develop efficient methods for solving the transshipment problem with mixed constraints. In this regard, we have searched some directions which may be helpful for developing methods for solving the proposed problem. They are as follows:

- **Primal Method**

  To solve transshipment problem with mixed constraints, we will try to convert the problem into equivalent transportation problem with mixed constraints and then method for finding the optimal solution will be developed.

- **Dual Method**

  We will try to convert the transshipment problem with mixed constraints into corresponding dual problem on the basis of conversion of primal linear programming problem and dual linear programming problem. In this approach the number of variables reduces. Afterwards attempt will be made to solve the dual problem.

- **Fourier Elimination Method**

  On the basis of the idea used in papers [19,9,22], for solving linear programming problem and transportation problem, we will take an attempt to solve the transshipment problem with mixed constraints by eliminating variables x_{ij} one by one In this approach all the variables will be eliminated except the objective function z which will declare the optimal solution.

- **Zero Suffix Method**

  Zero suffix method for finding an optimal solution of a classical transportation problem is developed by Sudhakar et.al.[21]. Using this idea we will try to solve the transshipment problem with mixed constraints. In this approach least cost cell would be searched by subtracting row and column minima in the transshipment table.

- **More-for-less Solution Method**

  Optimal "more-for-less" solution of transportation problem with mixed constraints is obtained by Pandian[18]. Note that more-for-less solution means more supply on less cost. We are going to extend this idea to make useful for solving the transshipment problem with mixed constraints.

Algorithms will be developed for solving the transshipment problems with mixed constraints in the directions specified above. Efficiency of the algorithms will be tested through various numerical examples.

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